WESTMINSTER SCHOOL
THE CHALLENGE 2015

MATHEMATICS III

Wednesday 29th April 2015

Time allowed: 1 hour 30 minutes

You may not use a calculator for this paper.

All your working should be clearly shown.

You should attempt all the questions.

1 Rain gauge A collects 1·6 litres of water in three-quarters of an hour.
Rain gauge B collects 7 cubic centimetres of water in 12 seconds.
Which rain gauge collects water at a faster rate? Show clearly the working you do to decide.

2 The marks in a Mathematics exam are divided in the ratio 3 : 5 between calculator and non-calculator questions. The marks for the calculator questions are divided in the ratio 3 : 2 between arithmetic and algebra questions, and the marks for the non-calculator questions are divided in the ratio 1 : 2 between arithmetic and algebra questions. There are four more marks for calculator arithmetic questions than for non-calculator arithmetic questions. How many marks were there for algebra questions?

3 Lists of numbers are defined by the rule

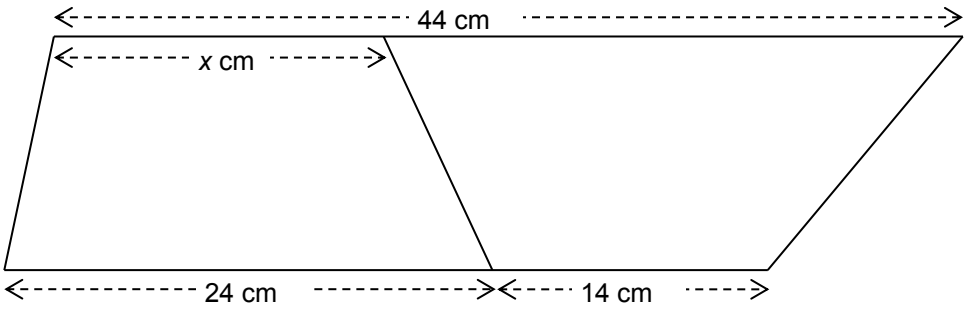
- if x is a number in the list, the next number in the list is $1-\frac{1}{x}$.

a If the first number in the list is 3, find
i the hundredth number in the list;
ii the sum of the first hundred numbers in the list.
b If the third number in the list is 5, what is the first number in the list?

4 The lowest common multiple of n, 16 and 36 is 432. List all the possible values of n.

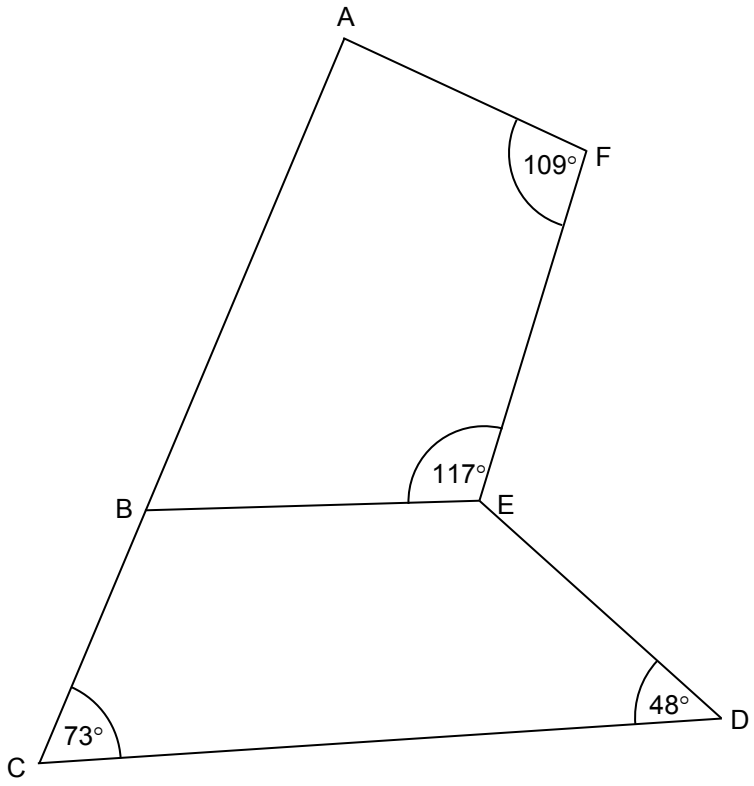
5 Ben, Sam and Tom are comparing their mathematics marks. Tom would have to increase his mark by 25% to equal Ben's mark. Sam would have to increase his mark by 40% to equal Ben's mark.
a By what percentage would Sam have to increase his mark to equal Tom's mark?
b If all the percentages in this question are exact, and the boys' mathematics marks were all whole numbers out of 50, what did each score?

6 The diagram shows two trapezia. The areas of the two trapezia are equal. What is length x?



7 There are eighteen boys and sixteen girls in a class. Twenty-two of these pupils have chosen football as their games option; the others have chosen hockey.
a How many more boys are there who have chosen football than girls who have chosen hockey?
b From the information given, you cannot tell how many girls have chosen football. What are the largest and smallest that this number could be, given the information you have?

8 In this diagram, ABC is a straight line.



Prove that ABEF and BCDE cannot both be trapeziums.

9 **a** P is a polygon with four sides, all of equal length.
For each of the following statements say whether it must be true, must be false, or can be true or false. Justify your claims carefully.

i P has four equal internal angles.
ii Any two opposite sides of P are parallel.
iii Exactly two of the internal angles of P are right angles.

b P is a polygon with six internal angles, all equal.
For each of the following statements say whether it must be true, must be false, or can be true or false. Justify your claims carefully.

i Each internal angle of P is 120°.
ii P is a regular hexagon.
iii Any two opposite sides of P are parallel.

10 At noon, two barrels of the same size are full of water. Each of them is leaking at a constant rate. The first takes 10 hours to empty completely, and the second takes 8 hours to empty completely.

a At what time is the volume of water left in the first barrel twice as great as the volume left in the second barrel?

b The next day, the first barrel is empty and the second is full. Water is added to each at a rate of 54 litres per hour, but continues to leak out of each at the same rates as before. The first barrel becomes full at exactly the same time as the second barrel becomes empty. What is the volume of the barrels?

11 **a** Alex, Basil, Chris and Dipak are comparing weights.

Alex is ten kilograms heavier than the average of the other three boys;
Basil is six kilograms heavier than the average of the other three boys;
Chris is two kilograms lighter than the average of the other three boys;
Dipak is fourteen kilograms lighter than the average of the other three boys.

The average weight of all four boys is 63·5 kg. What is the weight of each of them?

b Eric, Fred, Greg and Hasan are comparing weights.

Eric is thirteen kilograms heavier than the average of the other three boys;
Fred is four kilograms lighter than the average of the other three boys;
Greg is seven kilograms lighter than the average of the other three boys;

By how much is Hasan heavier or lighter than the average of the other three boys?

12 **a** A regular heptagon has vertices A, B, C, D, E, F and G, labelled in that order.

i Explain why AC, AD, AE and AF are diagonals of the heptagon, but AB and AG are not.

ii Use the diagram below to count how many diagonals a heptagon has. Remember that AD and DA are the same diagonal.

	A	**B**	**C**	**D**	**E**	**F**	**G**
A	✗	✗	✓	✓	✓	✓	✗
B	✗	✗	✗	✓	✓	✓	✓
C	✓	✗	✗	✗	✓	✓	✓
D	✓	✓	✗	✗	✗	✓	✓
E	✓	✓	✓	✗	✗	✗	✓
F	✓	✓	✓	✓	✗	✗	✗
G	✗	✓	✓	✓	✓	✗	✗

b **i** How many diagonals does a regular triskaidecagon have?
[Reminder: a triskaidecagon has 13 sides].

ii Find a formula for the number of diagonals in a regular polygon with n sides.

iii How many sides does a regular polygon with 560 diagonals have?

c Twenty married couples (forty players) play in a chess tournament. Every player plays every other player except their wife or husband once. How many games are played?

TURN OVER

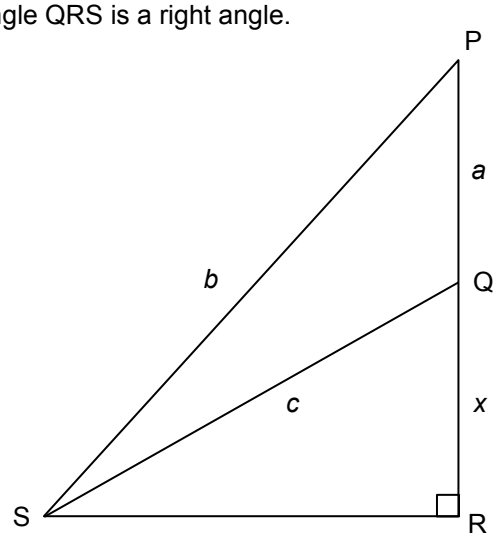
13 **a** **i** Multiply out

$$b^2 - (a + x)^2 .$$

ii Calculate

$$\frac{233^2 - 137^2}{240} .$$

In the diagram, angle QRS is a right angle.



b **i** Explain why

$$c^2 - x^2 = b^2 - (a + x)^2$$

ii Show how to derive the formula

$$x = \frac{b^2 - c^2 - a^2}{2a} .$$

iii Hence calculate x if $a = 120$, $b = 233$ and $c = 137$.

14 A Pythagorean triple is a set of three positive whole numbers a, b and c with the property

$$a^2 + b^2 = c^2 .$$

a Show that in a Pythagorean triple all three numbers can be even, but that it is not possible for all three numbers to be odd.

If a and c are odd, and b is even, define

$$p = \sqrt{\frac{c + a}{2}} \text{ and } q = \sqrt{\frac{c - a}{2}} .$$

b If $c = 65$ and $a = 33$, calculate b, p and q.

c If $p = 9$ and $q = 8$, find c and a.

d Find expressions for a and c in terms of p and q.

e Find an expression for b in terms of p and q.

f Find two square numbers that differ by 28^2.